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Combinatorial Thinking of College Students in Solving Distribution and Partition Type of a Combinatory Problem

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The objective of this study is to formulate the combinatorial thinking stage in college students in solving distribution and partition type of combinatorial problems. This study is a grounded theory. The subjects are the first year students of Department of Elementary School Teacher Education (PGSD) Universitas Muhammadiyah Surakarta, Indonesia. There are 40 (forty) college students where they are then reduced to 5 (five) students to be chosen as the research subjects by using the purposive sampling method. This method randomly chooses with a condition that the students already finished the Mathematical Basic Concept (for Elementary School) course. The result of this study is the 4 combinatorial thinking stages in solving the distribution and partition type of combinatorial problems, such as identifying, distributing/partitioning, concluding, and reflecting. Identification appears when the students learn the information in the question, understand the question given in the problem, and write the information of the problem. Distributing is the main characteristic of the distribution type of combinatorial problem, whereas partitioning is the main characteristic of the partition type of combinatorial problem. In this stage, students make possibilities of object order based on the criteria of the problem, distributing or dividing the object order based on the specified criteria, representing the answer in form of table and diagram, and also arranging, dividing, and choosing the information in the problem. Concluding is the third stage in combinatorial thinking in solving the selection type of combinatorial problem. The students in this stage count the amount of possible sets of order formed in the previous stage and making a conclusion from the given problem. The

fourth stage, reflecting, is where the students re-check the object order based on the criteria of the problem.

Key words: *Combinatorial thinking stage, Combinatorial problem, distribution and partition type.*

Introduction

Skills that need to be mastered in this 21st century are generally known as 7C, which are critical thinking, creativity, communication, collaboration, career and learning, self-dependent, cross-cultural understanding, and computing (Trilling, 2005; Widana, 2017). Critical thinking and creativity are parts of High Order Thinking Skills (HOTS) where in general, HOTS includes creative thinking skills, critical thinking, giving argument, logical thinking, reflective thinking, metacognitive, and decision-making skills. HOTS could help education members in producing problem-solving ideas (Turiman, et al., 2011; Heong, et al., 2012). HOTS is developed in elementary, middle, or even higher education stage. HOTS is introduced through learning and teaching activities in every subject including mathematics.

Muhsetyo (2008) stated that the mathematics subject is a process in giving learning experience to an education member through a series of planned activities so that the education member could obtain the competency of mathematics matters that have been given. Problem solving is a main part in all aspects of activities in mathematics learning. Nakin (2003) defined problem-solving as an effort to find solutions from a problem using knowledge, skills, and the understanding of the subject related to the problem.

One of the problems that can be used in problem-solving practice is a combinatory problem. Combination is one of the crucial parts in mathematics (English, 1991; Batanero, et al., 1997; NCTM, 2000). Berman & Fryer (1972) stated that the combinatorial problem is a study related to element stackings to be groups and also related to a general problem type in mathematics, which is enumeration. Combinatoric consists of structures which are rich in important mathematical principles and exists in many study areas including probability, computation, and division so that it has a crucial role in the mathematics curriculum even in the elementary school curriculum, to develop the thinking mental skill of the education member (English, 2005). This aspect is in accordance with a study by Lockwood (2013) who stated that the importance of combinatoric in kindergarten level until middle school and university curriculum has been present in the mathematics education literature. They have a big potential in the problem-solving context, and it's application in probability and computer science. Hence, the knowledge and pedagogy related combinatoric matter is important.

Logical thinking skills are highly needed to solve a combinatorial problem. Hidayat & Sumarmo (2013) defined logical thinking as a process to reach a conclusion using consistent reasoning, action and reaction thinking, thinking based on certain patterns, logical inference rules or logical principles to reach conclusions, and thinking which includes induction, deduction, analysis, and synthesis. Logical thinking definition is often switched with logical reasoning because both of them consist of similar activities. Basically, the logical thinking definition has a wider scope compared to that in logical reasoning. Tobin & Capie (1981) and Lay (2009) stated that measuring the logical thinking skill is based on the mental development theory from Piaget through the Test of Logical Thinking (TOLT), which consists of 5 components such as the controlling variable, proportional reasoning, probabilistics reasoning, correlational reasoning, and combinatorial thinking. Therefore, combinatorial thinking is one of the components in logical thinking.

Flavell (1962) mentioned that combinatorial thinking is a thinking method that includes all combination of possible objects, ideas, or propositions. Combinatorial thinking develops the knowledge of the education member to use a simple approach, which is by using the semiotic or sign approach (Godino et al., 2007). Tsai & Chang (2009) stated that combinatorial thinking triggers more creativity, curiosity, and confidence in the student when answering questions. Combinatorial thinking is a basic thinking skill that has to be consistently developed to reach a critical thinking skill.

Studies related to strategy in solving combinatorial problems has been done by several researchers (English, 1991; Eizenberg & Zaslavsky, 2004; Melusova & Vidermanova, 2015). English (1991) analysed six strategies to solve combinatorial problems. Eizenberg & Zaslavsky (2004) stated that a big obstacle faced by the education member in solving combinatorial problems is to find a correct solution and strategy to solve a problem. Melusova & Vidermanova (2015) studied about the methods used by the education member in solving a combinatorial problem. On the other hand, there is another study from Godino, et al. (2005) who studied three types of combinatorial problems such as selection, distribution, and partition type, and also the research from Lockwood (2013) who stated three models of combinatorial thinking such as formulas/expressions, counting processes, and sets of outcomes.

Hidayati, et al. (2019) stated that there are four combinatorial thinking stages of college students in solving combinatorial problems, such as identifying, selecting, concluding, and reflecting. This study will formulate student's combinatorial thinking stages in partition and distribution type of combinatorial problems. The distribution type of combinatorial problem has a keyword of "putting or placing", "introducing", "assigning", or "saving". Partition type combinatorial problem has keywords " giving", "dividing", or "grouping".

The new aspect in this study is the combinatorial thinking stage in solving partition and distribution type of combinatorial problems. The urgency in this study is to be used in identifying student's difficulty in solving combinatorial problems with the distribution and partition type.

Theory Discussion

Research Method

This study is a grounded theory research, because it is in line with the characteristic in it, which is theoretical sampling, the constant-comparative method, and specific ways of coding (Lichtman, 2009:73). Grounded theory focuses on data coding. There are three data coding processes in this study, such as (1) open coding, reading literature and gathering several categories which are relevant to the combinatorial thinking. (2) Axial coding, formulating a variety of the first assumptions in combinatorial thinking. (3) Selective coding, arranging a predictor regarding the stages of combinatorial thinking in solving the distribution and partition type of combinatorial problems.

Research Subject

The subjects in this study are 40 students from the Department of Elementary School Teacher Education (PGSD), Universitas Muhammadiyah Surakarta, Indonesia. The reason in choosing the subjects is to gather as much information as possible from many sources which are used to formulate the combinatorial thinking stage in solving combinatorial problems, and also to reveal the indicators in combinatorial thinking of students to solve the combinatorial problem. From the 40 college students, 5 of them are chosen to be the research subjects by using purposive sampling, which is choosing randomly with a requirement that the students have already finished Mathematics Basic Concept (for Elementary School) course. Besides that, the choice is also based on the communication skills and the student's willingness to be chosen as the research subject.

Research Procedure

There are five steps in this study. The first step is giving a test instrument related to the combinatorial problem with the distribution and partition type for the students to be done. The test instrument used in this study can be seen in figure 1 and figure 2.

Figure 1. Distribution Type of Combinatorial Problem

Chacha wants to give her balls to her two friends, Aliya and Salsa

a. Specify the possible set to give the ball to her two friends with a condition that Chacha could give all balls to one of her friends!

b. How many solutions that can be done by Chacha to give the balls to her friends ?

Figure 2. Partition Type of Combinatorial Problem

Chacha wants to move four balls inside 5 smaller boxes.

a. Specify the possible set to put the balls in each box with a condition that 1 box can only be filled by one ball!

b. How many possible solutions to put the 4 balls into the boxes ?

The second step is that the researcher observes, taking a direct note in the field, and recording all of the activities of the research subject when finishing the distribution and partition type combinatorial problem. The third step is that the researcher starts to analyse the combinatorial thinking stages which appears in each subject based on either the direct observation or the recording.

The fourth step is to do data triangulation to confirm the analysis result, which is by doing interview. The interview type used is a semi-structured type, so the interview guidance was not created. Besides the interview, data reduction is also done which eliminates irrelevant data after the interview is done. The last step is to conclude the analysis result regarding the combinatorial thinking stages based on the observation, recordings, and interview; hence the data are gathered about the combinatorial thinking stages and the behaviour indicator in solving the distribution and partition type of combinatorial problem.

Results and Discussion

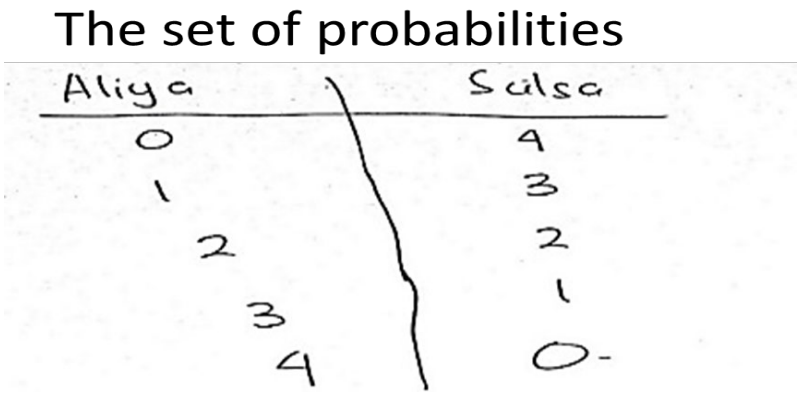
The subjects were given distribution, partition, and interview type of combinatorial problems. There were 10 students who could continuously trigger distribution and partition type of

combinatorial thinking steps. In solving the distribution type of combinatorial problem, the first step to do is that the subjects need to understand what the information given in the question are, understand the question, and write down the information given in the question. These steps are shown in the interview below:

S: *Similar to the previous question, when given this type of question, the first thing that I would do is to repeatedly reading the question until I understand the information given in the question, what the meaning of the question is, and then writing down the known information, the question, and the answer.*

The second step is that the subject makes a set of possibilities based on the question's criteria, distributing the object order based on the given criteria, representing the answer in a table, and arranging, sorting, and choosing the information given in the question. This step is illustrated in the subject's answer result below:

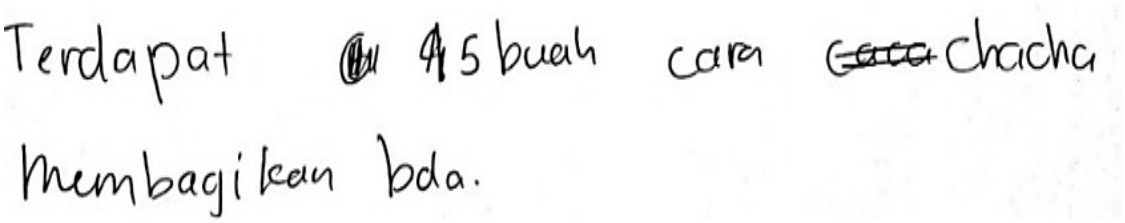
Figure 3. Answer sample from research subject about the object order in solving the distribution type of combinatorial problem



The third step is that the subject counts the amount of probability formed in the previous step and makes a conclusion from the given problem. This step is illustrated below, based on interview and the answer of the subject.

S: *I counted all the probabilities that I have written.*

Figure 4. Conclusion in solving the distribution type of combinatorial problem.



"There are 5 probabilities of Chacha distributing the balls."

The fourth step is that the subject re-checks the object arrangement result based on the problem's criteria. This aspect based on the interview is shown below.

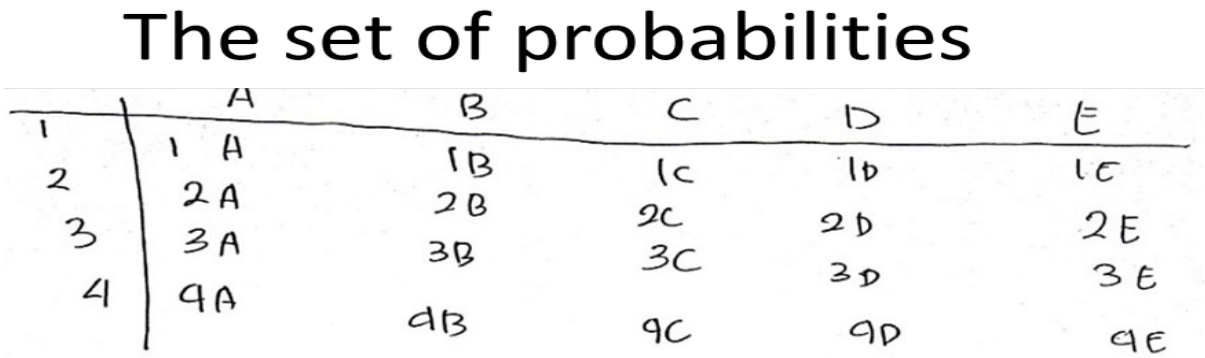
S: *After the answer is found, I check it again from the start, Ma'am. Is this correct?*

When the subject finished the partition type of combinatorial problem, the first step to do is learning the given information in the question, understanding the question of the problem, and writing the information of the problem. This aspect is shown in the interview below.

S: It is similar to the previous question, Ma'am. If I received that type of question, I would re-read it many times until I understand the information given in the question, what the question is about, then I write down the known information, the question, and the answer.

The second step is that the subject makes a set of possibilities based on the question's criteria, parting the object order based on the given criteria, representing the answer in a table, and arranging, sorting, and choosing the information given in the question. This step is illustrated in the subject's answer result below:

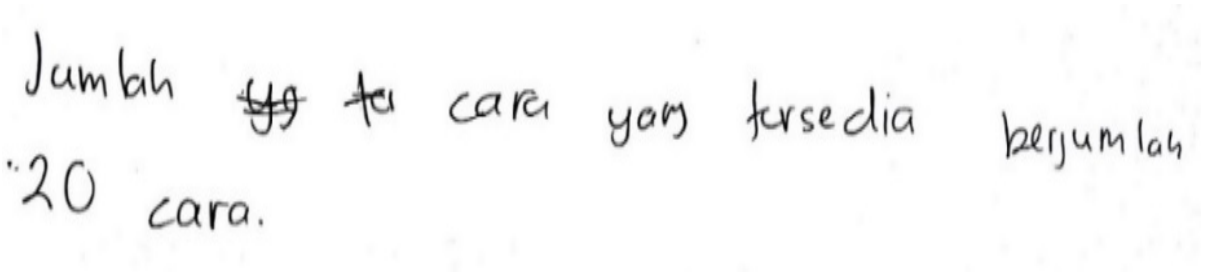
Figure 5. Answer sample from research subject about the object order in solving the partition type of combinatorial problem.



The third step is that the subject counts the amount of probability formed in the previous step and making a conclusion from the given problem. This step is illustrated below, based on the interview and the answer of the subject.

S: I counted all the probabilities, Ma'am. It is similar to the previous question.

Figure 6. Conclusion in solving the partition type of combinatorial problem.



"*The amount of the option probabilities are 20 options."*

The fourth step is that the subject re-check the object arrangement result based on the problem's criteria. This aspect based on the interview is shown below.

S: I doubted my answers, Ma'am, so I check it again like the previous question

Based on this research, it is shown that when the subject finished the distribution and partition type of combinatorial problem, there are 4 solution steps such as (1) understanding the given information in the question, understanding the question of the problem set and writing down the information given in the problem; (2) making a probability set based on the criteria of the problem, distributing and dividing the object order based on the specified criteria, representing the answers in a table, arranging, sorting, and choosing the information in the problem; (3) counting the amount of the probabilities of the order formed in the previous step and pulling a conclusion from the given problem; (4) rechecking the object order based on the criteria of the problem.

The difference exists in the second step. When the subject finished the distribution type of combinatorial problem, the second step in solving this problem is called "distributing" because the subject distributes the object order based on the criteria. This is shown from a student's answer, which is that when Aliya does not receive any ball then Salsa obtains 4 (four) balls, when Aliya gets 1 (one) ball then Salsa would get 3 (three) balls, Aliya receives 2 (two) balls then Salsa also get 2 (two) balls, Aliya gets 3 (three) balls then Salsa gets only 1 (one) balls, and when Aliya gets 4 (four) balls then Salsa does not receive any balls. The answers show the probabilities in distributing the balls based on the problem (Talebi & Nejad, 2019).

When the subject has finished the partition type of combinatorial problem, the second step is called parting because the subject divides the object order based on the criteria. It is shown based on the student's answer who makes the probality order in dividing 4 (four) balls to 5 other smaller boxes. Each ball has a probality to be put in the first, second, third, fourth, and fifth box.

These four steps are based on the 4 steps in solving the problem according to Polya, such as (1) the understanding of the problem; (2) planning of the problem solution; (3) doing the problem solving plan; and (4) rechecking the solution. On the other hand, Godino, et al. (2005) stated that the 5 indicators that a person can be said to think combinatorically are (1) understanding the problem correctly; (2) changing the problem into a mathematic symbol (linguistic elements); (3) making a problem-solving strategy (actions); (4) making the conclusion (definition and properties); and (5) making an explanation of the obtained conclusion (arguments). Based on the opinion of Polya and Godino,et al. (2005), the combinatorial thinking stages consists of 4 stages. The first stage is the stage of understanding the combinatorial problem, which is by understanding the given information in the question, understanding the question in the problem, and writing down the information in the problem. This stage could be defined as the information identification or identification stage.

The second stage is making the set of probabilities that could happen based on the criteria, distributing the object order based on the specified criteria, representing the answer in a table, arranging, sorting, and choosing the information given in the problem. In this second stage, making the possible set is the effort to plan the problem-solving steps consisting of changing the problem into mathematical symbols and making the problem-solving strategy. In the combinatorial problem, there are 2 types of problem which are distribution and partition so that in this second stage, there are 2 activities such as distributing and parting. The third stage counts the possible sets of answers that are made in the previous step and making a conclusion from the given problem or "concluding". The fourth step is rechecking the object order result based on the problem criteria, which is called "reflecting". The indicator of distribution and partition type of combinatorial problem can be seen in table 1 below.

Table 1: Combinatorial Thinking Stages in Solving Distribution and Partition type of Combinatorial Problem.

Combinatorial Thinking Stage	**Behaviour Indicator**
Identifying	1. Subject understands the given information in the question. 2. Subject understands the question in the problem and writes the information given in the problem.
Distributing and Partitioning	1. Subject makes the probability of possible order based on the criteria of the problem. 2. Subject distributes or parts the object order based on the specified criteria. 3. Subject represents the answers in the form of a table and diagram. 4. Subject arranges, sorts, and chooses the information in the problem.
Concluding	1. Subject counts the amount of order possibilities formed in the previous stage. 2. Subject make a conclusion from the given problem.
Reflecting	1. Subject rechecks the object order based on the criteria of the problem.

Based on the Table 1, the subject in the identification stage understands what the information in the question is, understands the question in the problem, and writes the information in the problem. The combinatorial problem used in this research is the distribution and partition type. Godino (2005) stated that a combinatorial problem is divided into 4, which are selection, distribution, partition, and the combination of distribution and partition.

The subject in the distribution and partition part are able to choose the object order based on the criteria and arranging the possible object based on criteria. The order could be represented using a table, diagram, list, and schemes (Godino et al., 2005). English (2005) stated that the usage of a diagram tree, systematical list, and table are the basic procedures in solving the combinatorial problem.

The subject in the conclusion stage, based on the existing problem, is able to count the amount of the probability of the order formed in the previous stage and make a conclusion based on the given problem. The implementation of this stage is related to the success in solving problems (Eizenberg & Zaslavsky, 2004). The subject in the reflecting stage is able to re-check the object order result based on the criteria of the problem. This situation is in line with the research of Batanero et. al. (1997) who stated that after a formal operational period, the teenagers are supposed to be able to find the procedure of systematical combinatorial

construction. The subjects in this research, the college students, are included in the stage after the formal operational period.

Conclusion

Based on the study results, the conclusion is that there are 4 stages in combinatorial thinking in solving the distribution and partition type of combinatorial problem, such as identifying, distributing/partitioning, concluding, and reflecting. The first stage, "identifying" is the stage where the students understand the information in the question, understanding the question in the problem, and writing the information in the problem.

The second stage, distributing, is the main characteristic of the distribution type of combinatorial problem, whereas the parting is the main characteristic in the partition type of combinatorial problem. In this stage, students make possibilities of object order based on the criteria of the problem, distributing or dividing the object order based on the specified criteria, representing the answer in the form of a table and diagram, and also arranging, dividing, and choosing the information in the problem.

Concluding is the third stage in combinatorial thinking in solving the selection type of combinatorial problem. The students in this stage count the amount of possible sets of order formed in the previous stage and make a conclusion from the given problem. The fourth stage, reflecting, is where the students re-check the object order based on the criteria of the problem.

The study result regarding the combinatorial thinking stages in solving distribution and partition type of combinatorial problem could identify the difficulties in the education member. The combinatorial thinking stages in solving the distribution and partition type of combinatorial problem could help the education member in solving the combinatorial problem, especially the distribution and partition types.

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